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Survival probability for brittle honeycombs under in-plane biaxial loading

JONG-SHIN HUANG*, CHONG-YI CHOU
Department of Civil Engineering, National Cheng Kung University, Tainan, 70101 Taiwan
E-mail: jshuang@mail.ncku.edu.tw

The failure envelopes of brittle honeycombs are affected by the cell-wall modulus of rupture. The variability in the cell-wall modulus of rupture is accounted for by assuming that it follows a Weibull distribution, giving the corresponding modulus of rupture for a prescribed survival probability. Furthermore, the existing model for the failure envelopes of honeycombs under in-plane biaxial loading is modified to take into account the effect of variability in the cell-wall modulus of rupture. Consequently, the failure envelopes of brittle honeycombs with a prescribed survival probability are developed. The effects of cell size, Weibull modulus and prescribed survival probability on the failure envelopes of brittle honeycombs are also evaluated.

1. Introduction

Cellular materials are increasingly used in many engineering applications such as thermal insulation, energy absorption and fire resistance. Meanwhile, cellular materials are typically used as lightweight cores in sandwich structures, especially for load-bearing components. In practice, ceramic cellular cores are preferable because of their high creep resistance and low thermal conductivity. For example, sandwich panels with a cement foam core and gypsum faces are widely utilized in building construction. However, pre-existing flaws within ceramic cellular materials resulting mainly from processing or machining might reduce their loading capacity and thus limit their application. At the same time, the tensile and compressive strengths of ceramic cellular materials are significantly influenced by the flaw size distribution within them. As a result, the strength variability in ceramic cellular materials is expected and can be observed from experiments.

In most cases, cellular materials in real engineering structures are subjected to a general multiaxial state of stress. Various failure mechanisms might occur for brittle cellular materials under multiaxial loading, depending on the properties of solid cell walls. Accounting for the cell-wall strength variability, brittle cellular materials could have different failure mechanisms and resulting failure envelopes which are essential to the designer. Therefore, the failure envelopes for brittle cellular materials under multiaxial loading need to be fully exploited in order to suggest ways of enhancing micro-structural design and material selection for lightweight structures. Here, we aim at analyzing the failure envelopes for brittle honeycombs under in-plane biaxial loading due to the simple, repeated and regular cell geometry in honeycombs. The results can provide a guideline in analyzing the multiaxial failure envelopes for brittle foams with more complicated cell geometry.

Gibson *et al*. [1] observed that bending moment dominates cell-wall deformation in honeycombs. Based on their cell-wall bending model, the in-plane linear elastic properties of honeycombs were calculated and found to be consistent with experimental measurements. For honeycombs in uniaxial compression, cell walls fail either by elastic buckling when the maximum compressive stress exceeds their Euler buckling load or by crushing when the maximum tensile stress exceeds their modulus of rupture [1–8]. By assuming a constant cell-wall modulus of rupture, the compressive crushing strength of honeycombs was described well by the cell-wall bending model proposed by Gibson and Ashby [9]. Elastic-buckling modes and combined elasto-plastic crushing for honeycombs were derived by Klintworth and Stronge [10]. For honeycombs in uniaxial tension, pre-existing cracks might cause catastrophic failure at a tensile stress much less than the compressive crushing strength. Maiti *et al*. [11] derived the expression for mode I fracture toughness of honeycombs based on the near-tip singular tensile stress of a continuum model. They found that the fast fracture strength of honeycombs depends on cell size, relative density, and cell-wall modulus of rupture.

When honeycombs are under in-plane biaxial loading, various failure mechanisms including elastic buckling, plastic yield and fast fracture were studied by Gibson *et al*. [12]. They developed equations describing the failure surfaces for honeycombs under in-plane biaxial loading. Huang and Lin [13] analyzed the mixed mode fracture for honeycombs under a combined loading of uniform tensile and in-plane shear stresses, resulting in a linear mixed-mode fracture criterion. The above

* Author to whom all correspondence should be addressed.

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studies assumed a constant cell-wall modulus of rupture, suitable for ductile honeycombs. In fact, the cell-wall modulus of rupture in brittle honeycombs may vary from one specimen to another, exhibiting strength variability. The strength variability in the cell-wall modulus of rupture should be taken into account in developing the failure envelopes for brittle honeycombs under in-plane biaxial loading.

Weibull [14] proposed an empirical formulation with a simple statistic distribution to describe the strength variability in brittle materials such as concrete, wood and glass fiber. It was found that a brittle solid with a larger volume possesses a lower tensile strength than that with a smaller volume; the difference in tensile strengths depends on the brittleness of the solid, namely the Weibull modulus. Jayatilaka and Trustrum [15] verified that the empirical Weibull modulus is related to the properties of cracks and flaws size distribution within brittle solids. In addition, the survival probability of a brittle solid subjected to a non-uniform tensile stress can be calculated from the Weibull statistic analysis. For example, Jayatilaka [16] calculated the survival probability of a brittle solid beam under bending moment loading using the Weibull statistic analysis. Huang and Gibson [17] found that the cell-wall modulus of rupture of brittle cordierite honeycombs was described well by the Weibull statistic analysis with a Weibull modulus of 6. Also, the cell-wall modulus of rupture depended on the volume of the cell wall, leading to a cell size effect. A similar cell size effect on the mode II fracture toughness of brittle honeycombs was presented by Huang and Lin [13].

In this paper, the strength variability in the cell-wall modulus of rupture is first studied using the Weibull statistic analysis. Then, the failure surfaces characterizing brittle crushing and fast brittle fracture for brittle honeycombs with a prescribed survival probability are developed. Finally, the failure envelopes of brittle honeycombs are plotted for various prescribed survival probabilities, cell sizes and Weibull moduli.

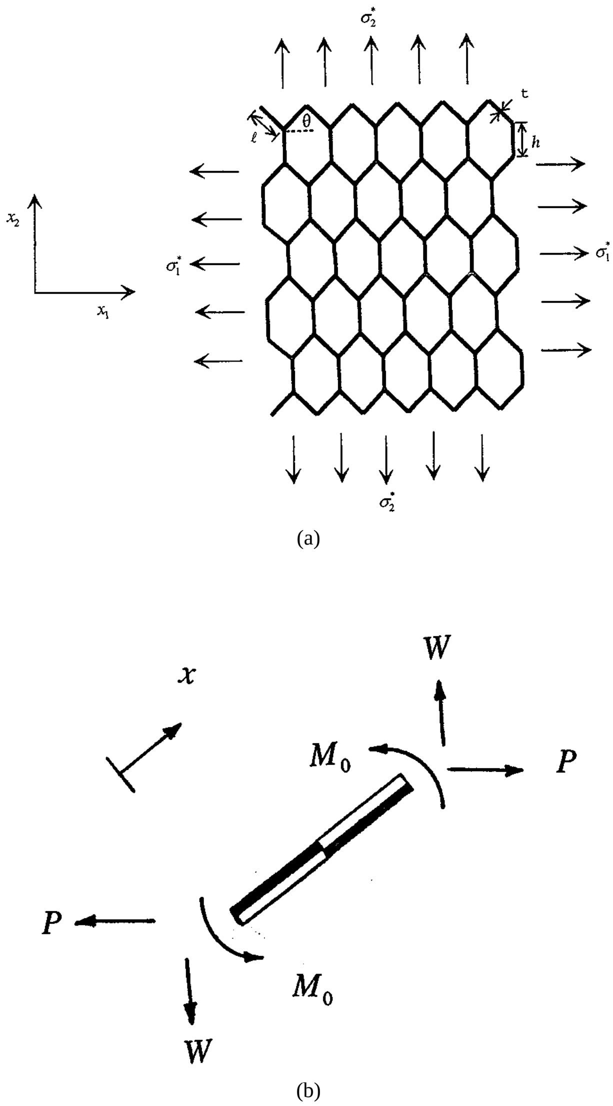

Figure 1 (a) An infinite honeycomb plate with a cell wall thickness t, an inclined cell length ℓ, a vertical cell length h, a cell angle θ and a honeycomb width b under in-plane biaxial loading. (b) The forces and bending moments exerted on an individual inclined cell wall.

2. Survival probability of brittle solid cell walls

A typical honeycomb with a cell wall thickness t, an inclined cell length ℓ, a vertical cell length h, a cell angle θ and a honeycomb width b is shown in Fig. 1a. For simplicity, the principal stresses are assumed to be aligned with the x_1 and x_2 axes. When the honeycomb is subjected to in-plane biaxial remote stresses σ_1^* and σ_2^*, the forces and moments exerted on an individual inclined cell wall are shown in Fig. 1b. Equilibrium requires that $W = \sigma_2^* \ell b \cos\theta$, $P = \sigma_1^*(h + \ell\sin\theta)b$ and $M_o = (P\ell\sin\theta - W\ell\cos\theta)/2$. The external moment exerted on any cross-section at a distance of x measured from one end of the solid cell wall beam is:

$$M = M_0 - P(\ell - x)\sin\theta + W(\ell - x)\cos\theta = (P\sin\theta - W\cos\theta)\left(x - \frac{\ell}{2}\right) \qquad (1)$$

The normal stress at a distance of y from the neutral axis of the solid cell wall beam can be calculated from the elementary mechanics of materials:

$$\sigma_n = \frac{My}{I} = \frac{12(P\sin\theta - W\cos\theta)\left(x - \frac{\ell}{2}\right)y}{bt^3} \qquad (2)$$

Here $I = bt^3/12$ is the moment of inertia of the solid cell wall beam.

For a brittle solid with a volume of V subjected to a non-uniform tensile stress, the failure probability of the solid can be calculated from the Weibull statistic analysis [14,16]:

$$P_f = 1 - \exp\left[-\int_V \left(\frac{\sigma_s}{\sigma_0}\right)^m \frac{dV}{V_0}\right] \qquad (3)$$

Here V_0 is a unit volume (namely, 1 cm^3 or 1 mm^3), σ_s is the normal tensile stress acting at any point within the solid, σ_0 is a scale parameter and m is the Weibull modulus. The magnitude of the Weibull modulus describing the brittleness of the solid depends on the flaw size distribution within it; a solid with a smaller m is more brittle. It should be noted that only tensile stresses within the solid are taken into account in calculating the failure probability of Equation 3.

In most honeycombs, the tensile stress of an individual inclined cell wall caused by bending moment is much larger than that either by shear or by axial forces. For simplicity, the failure probability of the solid cell-wall beam can be obtained by substituting Equation 2 into 3, giving:

$$\begin{aligned} P_f &= 1 - \exp\left\{-\int_V \left[\frac{12(P\sin\theta - W\cos\theta)}{bt^3\sigma_0} \times \left(x - \frac{\ell}{2}\right)y\right]^m\right\}\frac{dV}{V_0} \\ &= 1 - \exp\left\{-\frac{2b}{V_0}\left[\frac{12(P\sin\theta - W\cos\theta)}{bt^3\sigma_0}\right]^m \times \int_{x=\ell/2}^{\ell}\int_{y=0}^{t/2}\left(x - \frac{\ell}{2}\right)^m y^m\,dx\,dy\right\} \\ &= 1 - \exp\left\{-\frac{1}{2(m+1)^2}\left(\frac{V}{V_0}\right) \times \left[\frac{3(P\sin\theta - W\cos\theta)\ell}{bt^2\sigma_0}\right]^m\right\} \end{aligned} \quad (4)$$

Since $W = \sigma_2^*\ell b\cos\theta$ and $P = \sigma_1^*(h + \ell\sin\theta)b$, the above equation can be rewritten as:

$$P_f = 1 - \exp\left\{-\frac{1}{2(m+1)^2}\left(\frac{V}{V_0}\right)\left(3\frac{\ell^2}{t^2}\right)^m\left[(h/\ell + \sin\theta)\sin\theta\left(\frac{\sigma_1^*}{\sigma_0}\right) - \cos^2\theta\left(\frac{\sigma_2^*}{\sigma_0}\right)\right]^m\right\} \quad (5)$$

The maximum tensile stress of the solid cell-wall beam occurs at its both ends:

$$\sigma_{\max} = \frac{M_0(t/2)}{bt^3/12} = 3\frac{\ell^2}{t^2}\left[\sigma_1^*(h/\ell + \sin\theta)\sin\theta - \sigma_2^*\cos^2\theta\right] \quad (6)$$

By substituting Equation 6 into 5, the survival probability of the solid cell-wall beam can be expressed as:

$$P_s = 1 - P_f = \exp\left\{-\frac{1}{2(m+1)^2}\left(\frac{V}{V_0}\right)\left(\frac{\sigma_{\max}}{\sigma_0}\right)^m\right\} \quad (7)$$

When the maximum tensile stress reaches the cell-wall modulus of rupture σ_{fs}, failure occurs. By setting $\sigma_{\max} = \sigma_{fs}$ in Equation 7, the survival probability of the solid cell-wall beam becomes:

$$P_s = \exp\left\{-\frac{1}{2(m+1)^2}\left(\frac{V}{V_0}\right)\left(\frac{\sigma_{fs}}{\sigma_0}\right)^m\right\} \quad (8)$$

Therefore, the corresponding cell-wall modulus of rupture for a prescribed survival probability P_s is found to be:

$$\sigma_{fs} = \left[2(m+1)^2\left(\frac{V_0}{b\ell^2}\right)\left(\frac{\ell}{t}\right)\log\left(\frac{1}{P_s}\right)\right]^{1/m}\sigma_0 \quad (9)$$

Here the volume of the solid cell-wall beam is $V = \ell bt$. From Equation 9, it is known that the cell-wall modulus of rupture in brittle honeycombs is not a constant, depending on the cell-wall volume, the material parameters m and σ_0 of solid cell walls, and the prescribed survival probability.

The mean modulus of rupture of the solid cell wall beam can be calculated from Equation 8 and found to be:

$$\begin{aligned} \overline{\sigma_{fs}} &= \int_0^\infty P_s\,d\sigma_{fs} = \int_0^\infty \exp\left[\frac{-1}{2(m+1)^2}\left(\frac{V}{V_0}\right) \times \left(\frac{\sigma_{fs}}{\sigma_0}\right)^m\right]d\sigma_{fs} \\ &= \sigma_0\left[2(m+1)^2\left(\frac{V_0}{V}\right)\right]^{1/m}\Gamma\left(1 + \frac{1}{m}\right) \end{aligned} \quad (10)$$

Here $\Gamma(1 + m^{-1})$ is the gamma function. Then, the ratio of the cell-wall modulus of rupture for a prescribed survival probability and the mean cell-wall modulus of rupture is:

$$\frac{\sigma_{fs}}{\overline{\sigma_{fs}}} = \frac{[\log(1/P_s)]^{1/m}}{\Gamma(1 + 1/m)} \quad (11)$$

The ratio depends on the prescribed survival probability and the Weibull modulus of solid cell-wall materials.

3. Survival probability of brittle honeycombs

The existing model for the failure envelopes of honeycombs under in-plane biaxial loading [12] will be modified to take into account the effect of variability in the cell-wall modulus of rupture. For brittle honeycombs under in-plane biaxial loading, three failure mechanisms are possible and will be considered here: brittle crushing, fast brittle fracture and elastic buckling. The mechanism of failure for brittle honeycombs depends on stress state. Since the cell-wall modulus of rupture is not a constant, the failure envelopes of brittle honeycombs will be also affected by prescribed survival probability and Weibull modulus of solid cell walls.

3.1. Brittle crushing

When a brittle honeycomb is under in-plane biaxial loading, the axial force acting on any cross-section of an individual inclined cell wall is $N = -(P\cos\theta + W\sin\theta)$. Then, the resulting uniform axial stress is:

$$\begin{aligned} \sigma_a &= \frac{N}{bt} \\ &= -\left[\sigma_1^*(h/\ell + \sin\theta)\cos\theta + \sigma_2^*\cos\theta\sin\theta\right]\left(\frac{\ell}{t}\right) \end{aligned} \quad (12)$$

The critical skin stress occurs at both ends of the cell wall, contributed from both bending moment and axial force; the effect of shear force is negligible. Brittle crushing occurs when the critical skin tensile stress reaches the cell-wall modulus of rupture for a given survival probability. Hence, the maximum tensile stress resulting from bending moment at failure is $\sigma_{fs} - \sigma_a$; the bending moment can be either positive or negative. From Equations 6, 9 and 12, the failure remote stresses for brittle crushing are found to be:

$$\begin{aligned} &\pm 3\left(\frac{\ell}{t}\right)^2 \left[\sigma_1^*(h/\ell + \sin\theta)\sin\theta - \sigma_2^*\cos^2\theta\right] \\ &\quad + \left(\frac{\ell}{t}\right)\left[\sigma_1^*(h/\ell + \sin\theta)\cos\theta + \sigma_2^*\cos\theta\sin\theta\right] \\ &= \sigma_0\left[2(m+1)^2\frac{V_0}{b\ell^2}\left(\frac{\ell}{t}\right)\log\left(\frac{1}{P_s}\right)\right]^{1/m} \\ &= \overline{\sigma_{fs}}\frac{[\log(1/P_s)]^{1/m}}{\Gamma(1+1/m)} \end{aligned} \quad (13)$$

The relative density of the honeycomb (the density of the honeycomb divided by that of the solid from which it is made) is proportional to the ratio of the cell wall thickness to length, t/ℓ. From Equation 13, it is known that the failure remote stresses for brittle crushing depend on the cell size, relative density and Weibull modulus of the honeycomb, and the prescribed survival probability.

3.2. Fast brittle fracture

Fig. 2 illustrates a brittle honeycomb plate with a central macro-crack, c, subjected to a uniform remote tensile stress σ_2^* in the x_2 direction. The tensile strength of the brittle honeycomb will be much lower than its brittle crushing strength due to the stress concentration effect around the crack tip. The continuum model proposed by Maiti *et al.* [11] is utilized here to calculate the tensile strength of the brittle honeycomb. A singular stress field develops ahead of the crack tip, giving a local stress of:

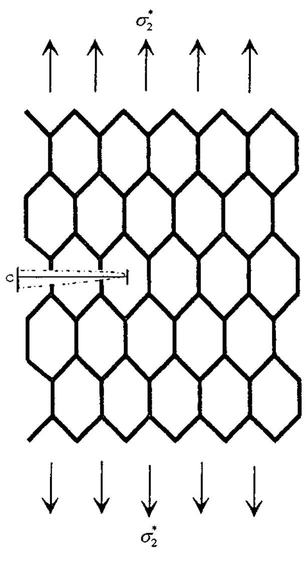

Figure 2 An infinite brittle honeycomb plate with a central macrocrack c under a uniform remote tensile stress σ_2^* in the x_2 direction.

$$\sigma_{\text{Local}} = \frac{\sigma_2^*\sqrt{\pi c}}{\sqrt{2\pi r}} \quad (14)$$

where r is the distance ahead of the crack tip. The bending moment exerted on the first unbroken cell wall along the macrocrack plane is then:

$$\begin{aligned} M_2 &= \int_0^{\ell\cos\theta} \sigma_{\text{Local}} b(\ell\cos\theta - r)dr \\ &= \frac{4}{3\sqrt{2}}\sigma_2^* b\sqrt{c}(\ell\cos\theta)^{3/2} \end{aligned} \quad (15)$$

The critical tensile stress on the first unbroken cell wall is:

$$\sigma_{\text{critical}} = \frac{M_2(t/2)}{bt^3/12} = \frac{4\sqrt{2}\sigma_{21}^*(\ell\cos\theta)^{3/2}\sqrt{c}}{t^2} \quad (16)$$

When the critical tensile stress exceeds the cell-wall modulus of rupture for a prescribed survival probability, the first unbroken cell wall fractures and the macrocrack advances one cell size distance, giving the remote stress σ_2^* at failure:

$$\begin{aligned} \sigma_2^* &= \sigma_0\left[2(m+1)^2\left(\frac{V_0}{b\ell^2}\right)\left(\frac{\ell}{t}\right)\log\left(\frac{1}{P_s}\right)\right]^{1/m} \\ &\quad \times \frac{1}{4\sqrt{2}\cos^{3/2}\theta}\left(\frac{t}{\ell}\right)^2\sqrt{\frac{\ell}{c}} \\ &= \overline{\sigma_{fs}}\frac{[\log(1/P_s)]^{1/m}}{\Gamma(1+1/m)}\frac{1}{4\sqrt{2}\cos^{3/2}\theta}\left(\frac{t}{\ell}\right)^2\sqrt{\frac{\ell}{c}} \end{aligned} \quad (17)$$

Similarly, the remote stress σ_1^* at failure in the x_1 direction can be obtained:

$$\begin{aligned} \sigma_1^* &= \sigma_0\left[2(m+1)^2\left(\frac{V_0}{b\ell^2}\right)\left(\frac{\ell}{t}\right)\log\left(\frac{1}{P_s}\right)\right]^{1/m} \\ &\quad \times \frac{1}{2(h/\ell+\sin\theta)^{3/2}}\left(\frac{t}{\ell}\right)^2\sqrt{\frac{\ell}{c}} \\ &= \overline{\sigma_{fs}}\frac{[\log(1/P_s)]^{1/m}}{\Gamma(1+1/m)}\frac{1}{2(h/\ell+\sin\theta)^{3/2}} \\ &\quad \times\left(\frac{t}{\ell}\right)^2\sqrt{\frac{\ell}{c}} \end{aligned} \quad (18)$$

Again, the remote stresses σ_1^* and σ_2^* at failure depend on the Weibull modulus, cell size, relative density of the honeycombs, and the prescribed survival probability.

3.3. Elastic buckling

Brittle honeycombs under either uniaxial or biaxial compressive loading might fail due to the bucking of one set of cell walls loaded axially up to their Euler buckling load. The biaxial remote stresses in Fig. 1a produce an axial load on the vertical cell wall of $2\sigma_2^*\ell b\cos\theta$. When the axial load reaches the Euler

TABLE I End Constraint factor n^2 for elastic buckling of honeycombs [12]

σ_1^*/σ_2^*	n^2 (mode 1)	n^2 (mode 2)
0	0.44	—
1/3	0.419	0.648
1/2	0.407	0.547
1	0.370	0.370
2	0.306	0.222
3	0.269	0.156

buckling load, elastic buckling occurs, giving the buckling strength of the honeycomb:

$$\sigma^*_{2,\text{Buckling}} = \frac{n^2\pi^2 E_s t^3}{24\ell h^2 \cos\theta} = \frac{n^2\pi^2}{24(h/\ell)^2\cos\theta} E_s \left(\frac{t}{\ell}\right)^3 \tag{19}$$

Here E_s is the elastic modulus of solid cell walls. End constraint factor n^2 depends on stress state and buckling mode; Gibson *et al*. [12] presented a full analysis for two possible buckling modes and the corresponding end constraint factor for various biaxial stress states.

Since the Euler buckling load depends only on the elastic modulus and slenderness of the individual cell walls, the buckling strength of the honeycomb will not be affected by the Weibull modulus of solid cell walls and the prescribed survival probability. The end constraint factors for regular hexagonal honeycombs under biaxial loading suggested by Gibson *et al*. [12] are listed in Table I and will be utilized to construct the failure envelopes for brittle honeycombs.

4. Discussions

Based on the above analysis, it is found that the cell-wall modulus of rupture in brittle honeycombs depends on the volume and Weibull modulus of solid cell walls, and the prescribed survival probability. Consider two brittle honeycombs made from the same solid material but with different cell size, relative density and prescribed survival probability; ℓ_1, t_1/ℓ_1 and $P_{s,1}$ for honeycomb 1 while ℓ_2, t_2/ℓ_2 and $P_{s,2}$ for honeycomb 2. From Equation 9, the ratio of the cell-wall moduli of rupture for the two honeycombs is found to be:

$$\frac{\sigma_{fs,1}}{\sigma_{fs,2}} = \left[\left(\frac{b\ell_2 t_2}{b\ell_1 t_1}\right)\log\left(\frac{P_{s,2}}{P_{s,1}}\right)\right]^{1/m} = \left[\left(\frac{\ell_2}{\ell_1}\right)^2\left(\frac{t_2/\ell_2}{t_1/\ell_1}\right)\log\left(\frac{P_{s,2}}{P_{s,1}}\right)\right]^{1/m} \tag{20}$$

Both honeycombs have the same width of b. It is noted that the cell-wall modulus of rupture increases with decreasing cell size, relative density and prescribed survival probability. In other words, brittle honeycombs with a lower prescribed survival probability, a smaller cell size and a lower relative density will possess a larger cell-wall modulus of rupture. Since m is larger than zero, the ratio in Equation 20 decreases when the Weibull modulus becomes larger. Equation 20 also indicates that the cell-wall modulus of rupture is a constant regardless of prescribed survival probability, cell size and relative density as the Weibull modulus approaching infinity for ductile honeycombs.

From Equations 13, 17, 18 and 19, it is known that the failure stresses for brittle crushing and fast brittle fracture in brittle honeycombs are affected by cell size, relative density, Weibull modulus and prescribed survival probability while those for elastic buckling are only influenced by relative density. To investigate the effect of prescribed survival probability on the failure surfaces for brittle honeycombs under in-plane biaxial loading, the Weibull modulus is first kept fixed and three different prescribed survival probabilities of 0.2, 0.5 and 0.8 are considered here. The resulting failure envelopes are plotted in Figs 3–6 for brittle honeycombs with a Weibull modulus of 2, 4, 8 and 100, respectively. In the figures, the cell geometry and material properties of brittle honeycombs are assumed to be: $h/\ell = 1$, $\theta = 30^\circ$, $c/\ell = 2$, $t/\ell = 0.1$ and $\overline{\sigma_{fs}}/E_s = 0.01$. From Figs 3–5, it is seen that the area contained within the failure envelopes for brittle honeycombs with a higher prescribed survival probability is smaller than that with a lower prescribed survival probability. For a brittle honeycomb with a higher prescribed survival probability, the cell-wall modulus of rupture will be smaller. As a result of that, the brittle honeycomb under a given biaxial stress state will be more likely to fail, giving a smaller area contained within the failure envelope.

From Figs 3–6, it is also seen that the difference between the areas contained within the failure envelopes for various survival probabilities becomes smaller as the Weibull modulus increases. That is, for a prescribed survival probability, the failure stresses for honeycombs with same cell size and relative density but with a smaller Weibull modulus will scatter more widely than those with a larger Weibull modulus. It is expected that the failure envelopes will come close to a set of intersecting lines when the Weibull modulus becomes much larger as shown in Fig. 6 for honeycombs with same Weibull modulus $m = 100$ but different prescribed survival probabilities.

Since the cell-wall modulus of rupture depends on its volume, the biaxial failure stresses are different for brittle honeycombs with various cell sizes even though they have same cell geometry and relative density. For instance, two brittle honeycombs have the same relative density, Weibull modulus and prescribed survival probability but different cell size; namely, $t_1/\ell_1 = t_2/\ell_2 = t/\ell, h_1/\ell_1 = h_2/\ell_2 = h/\ell, \theta_1 = \theta_2 = \theta, m_1 = m_2 = m$ and $P_{s,1} = P_{s,2}$ but $\ell_1 \neq \ell_2$. The failure stresses for brittle crushing in honeycomb 1 can be expressed in terms of the cell-wall modulus of rupture in honeycomb 2, σ_{fs}:

$$\pm 3\left(\frac{\ell}{t}\right)^2\left[\sigma_1^*(h/\ell + \sin\theta)\sin\theta - \sigma_2^*\cos^2\theta\right] + \left(\frac{\ell}{t}\right)\left[\sigma_1^*(h/\ell + \sin\theta)\cos\theta + \sigma_2^*\cos\theta\sin\theta\right] = \sigma_{fs}\left(\frac{\ell_2}{\ell_1}\right)^{2/m} \tag{21}$$

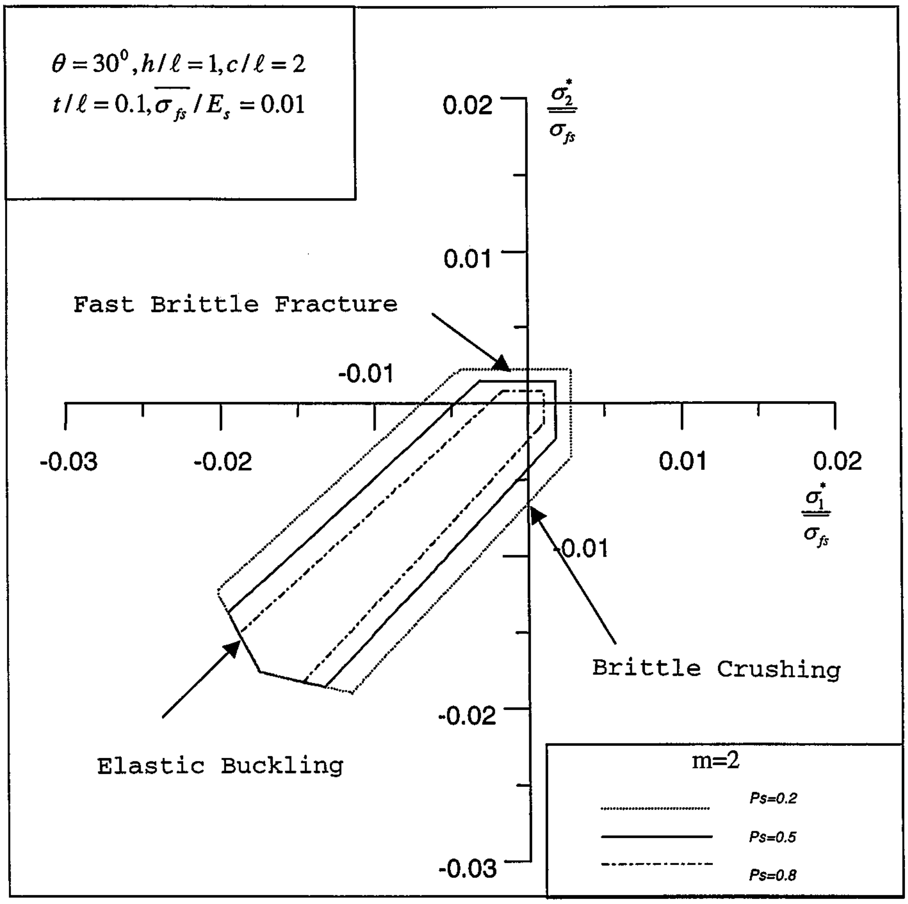

Figure 3 Failure envelopes for brittle honeycombs with $m = 2$ and various prescribed survival probabilities of 0.2, 0.5 and 0.8. The failure stresses for both brittle crushing and fast brittle fracture increases significantly with decreasing prescribed survival probability while the failure stresses for elastic buckling are not affected.

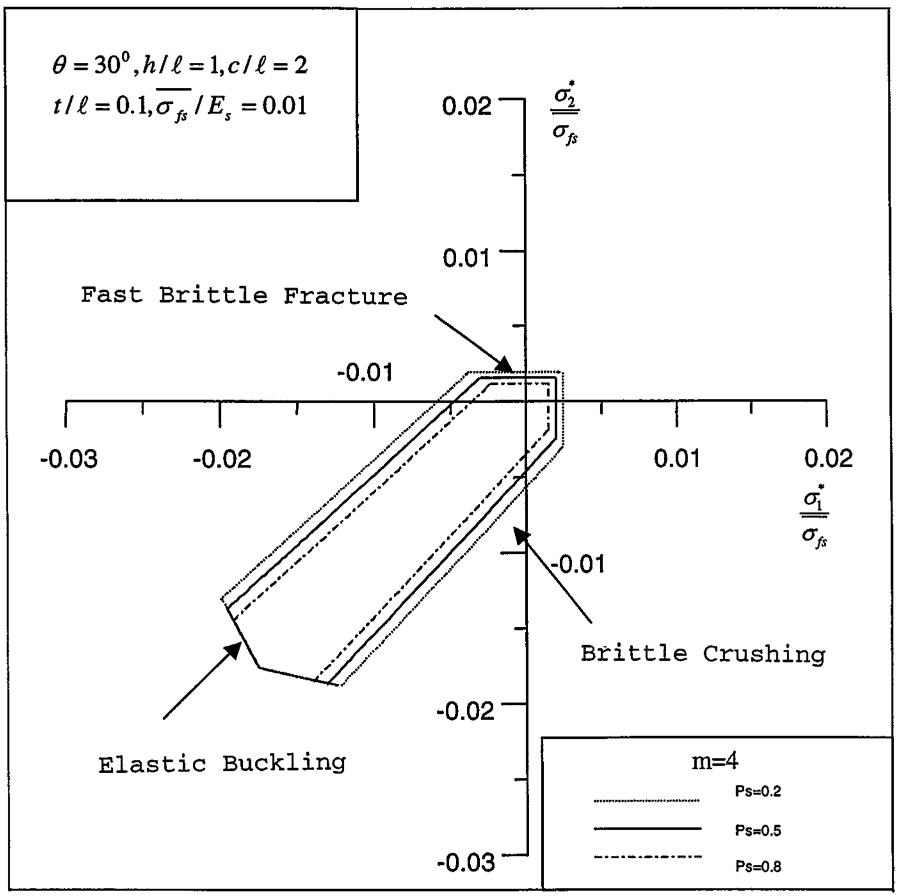

Figure 4 Failure envelopes for brittle honeycombs with $m = 4$ and various prescribed survival probabilities of 0.2, 0.5 and 0.8. The areas contained within the failure envelopes increase moderately with decreasing prescribed survival probability.

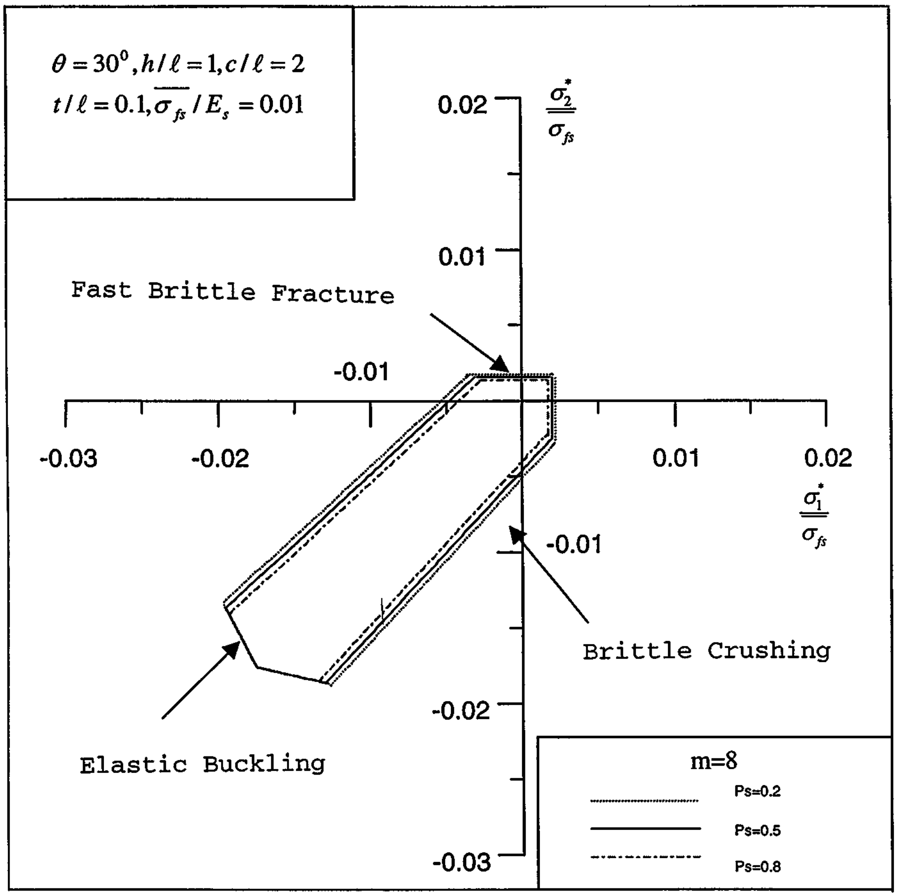

Figure 5 Failure envelopes for brittle honeycombs with $m = 8$ and various prescribed survival probabilities of 0.2, 0.5 and 0.8. The areas contained within the failure envelopes increase slightly with decreasing prescribed survival probability.

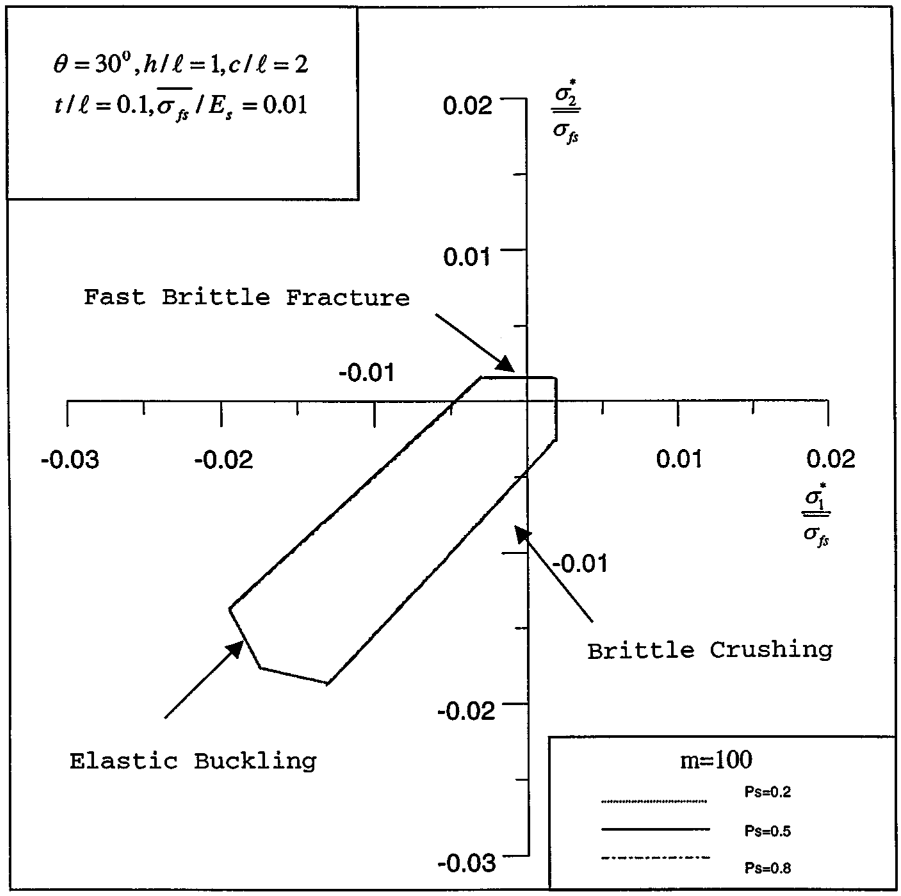

Figure 6 Failure envelopes for brittle honeycombs with $m = 100$ and various prescribed survival probabilities of 0.2, 0.5 and 0.8. The failure envelopes for various survival probabilities come closer to a set of intersecting lines as suggested by the existing model [12] for ductile honeycombs.

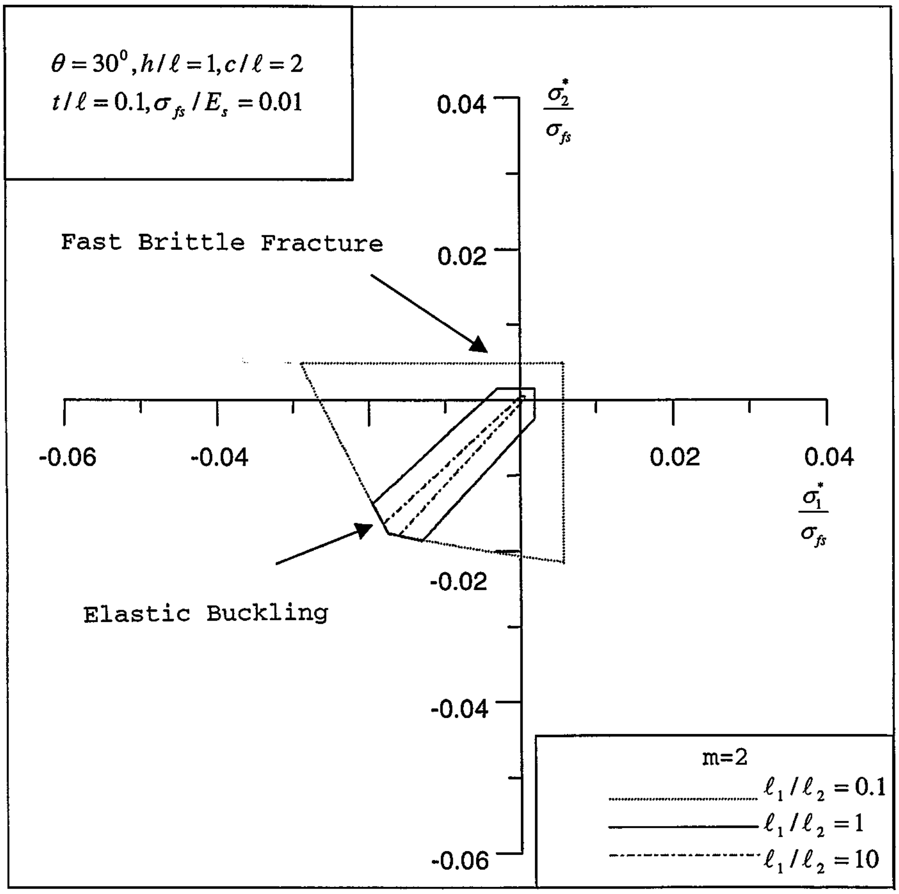

Figure 7 Failure envelopes for brittle honeycombs with $m = 2$ and different cell sizes. The failure stresses of fast brittle fracture and brittle crushing are higher for brittle honeycombs with a smaller cell size. There is no cell size effect for elastic buckling.

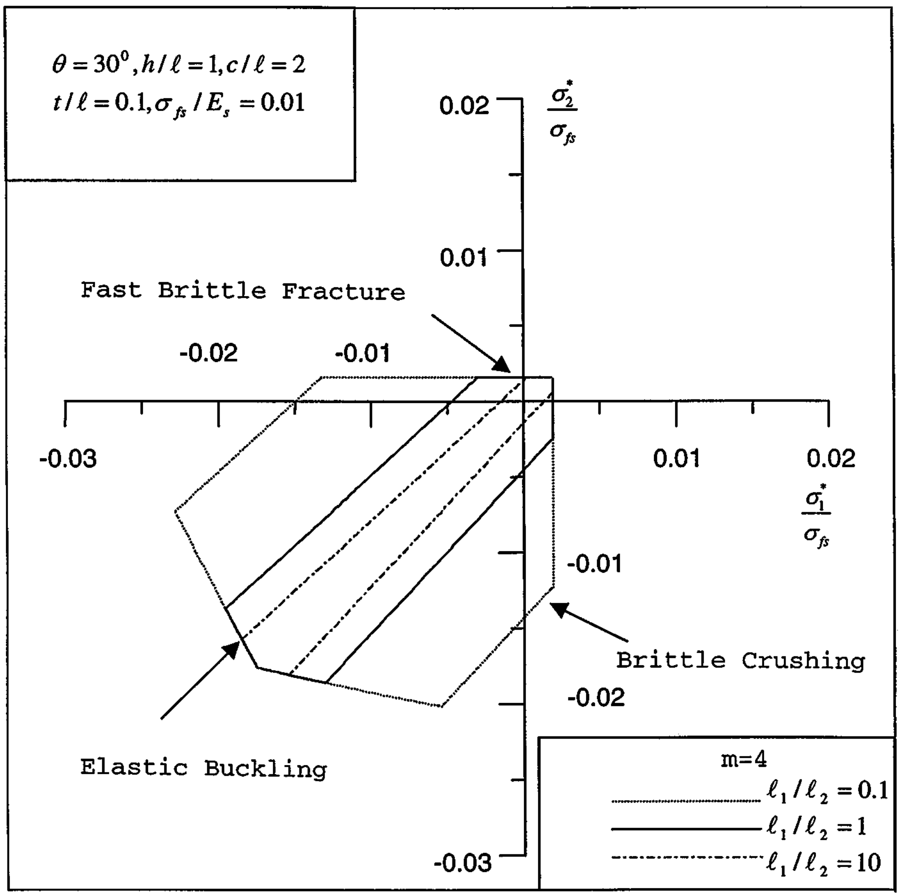

Figure 8 Failure envelopes for brittle honeycombs with $m = 4$ and different cell sizes. Brittle honeycombs with a smaller cell size have higher failure stresses for brittle crushing. There is no cell size effect for both elastic buckling and fast brittle fracture.

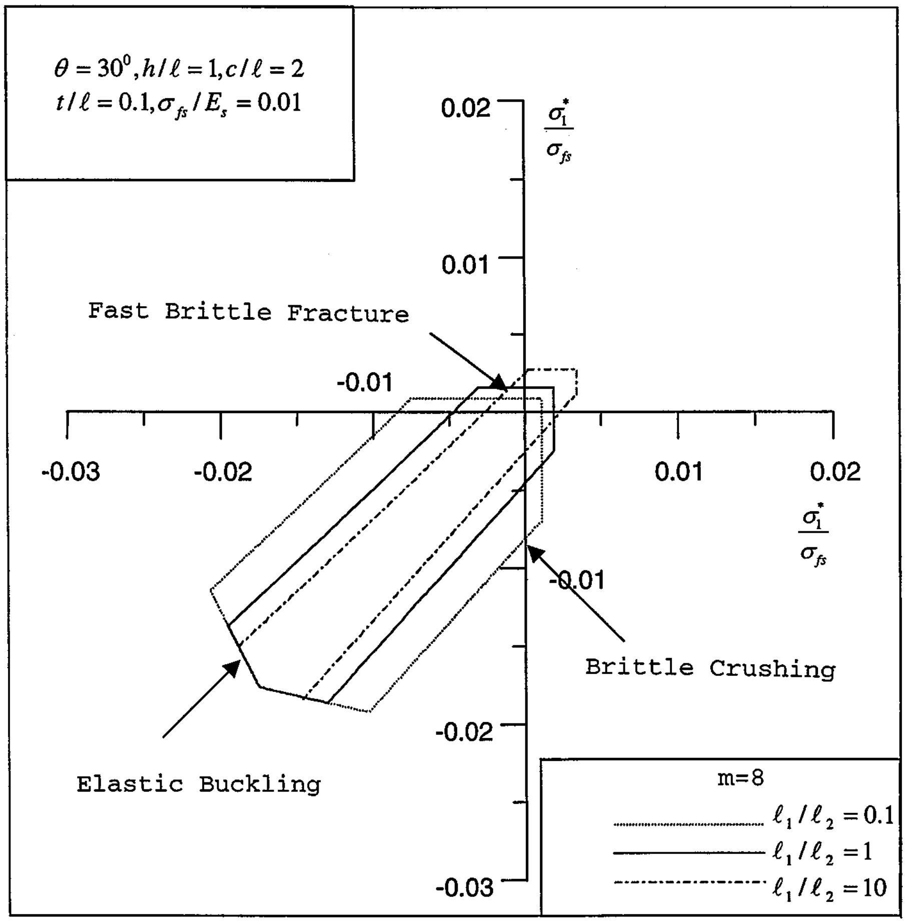

Figure 9 Failure envelopes for brittle honeycombs with $m = 8$ and different cell sizes. Brittle honeycombs with a smaller cell size have higher failure stresses for brittle crushing but lower failure stresses for fast brittle fracture. There is no cell size effect for elastic buckling.

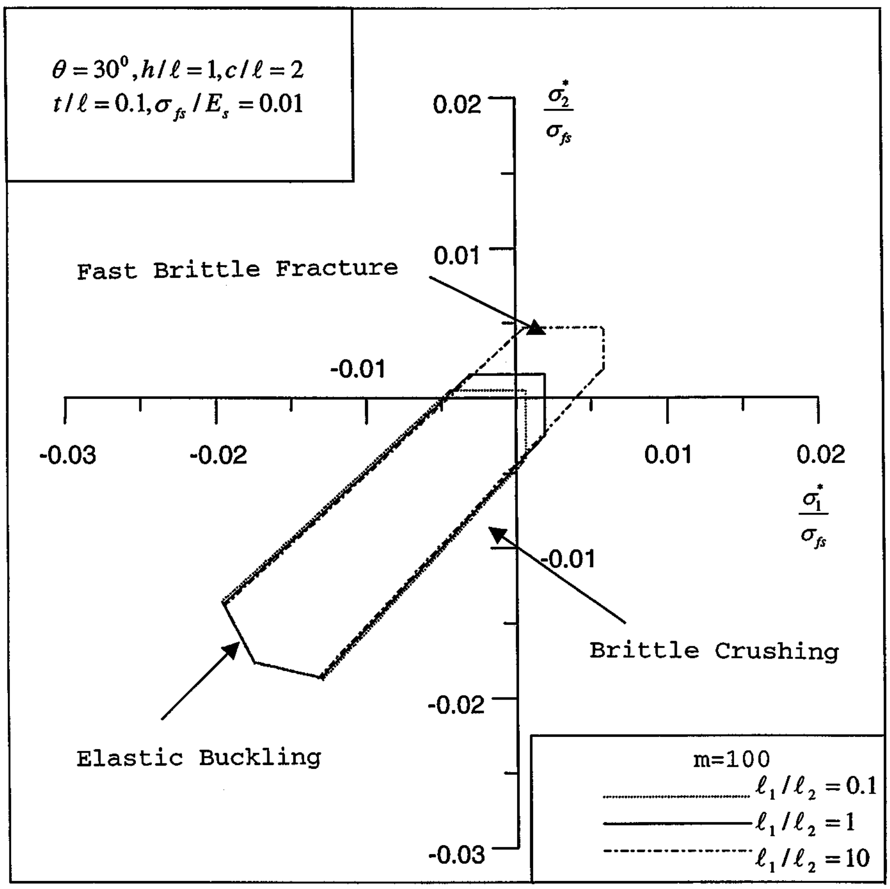

Figure 10 Failure envelopes for brittle honeycombs with $m = 100$ and different cell sizes. The failure stresses of fast brittle fracture is lower for brittle honeycombs with a smaller cell size. The cell size effect on brittle crushing and elastic buckling is negligible.

Meanwhile, if $c_1 = c_2 = c$ and $c/\ell_2 = c/\ell$, the failure stresses for fast brittle fracture in honeycomb 1 can also be rewritten as:

$$\frac{\sigma_1^*}{\sigma_{fs}} = \frac{1}{2(h/\ell + \sin\theta)^{3/2}}\left(\frac{t}{\ell}\right)^2 \sqrt{\frac{\ell}{c}}\left(\frac{\ell_2}{\ell_1}\right)^{2/m-1/2} \tag{22}$$

and

$$\frac{\sigma_2^*}{\sigma_{fs}} = \frac{1}{4\sqrt{2}\cos^{3/2}\theta}\left(\frac{t}{\ell}\right)^2 \sqrt{\frac{\ell}{c}}\left(\frac{\ell_2}{\ell_1}\right)^{2/m-1/2} \tag{23}$$

The Euler buckling load of solid cell walls, however, depends only on the relative density instead of the cell size of brittle honeycombs. As a result, the failure stresses for elastic buckling in brittle honeycombs exhibit no cell size effect.

The failure surfaces of individual mechanisms in brittle honeycombs with different cell sizes are plotted in Figs 7–10 for various Weibull moduli. In the figures, the cell size ratio ℓ_1/ℓ_2 is set to be 0.1, 1.0 and 10, and the cell geometry and material properties of the honeycombs are assumed to be $c/\ell = 2$, $h/\ell = 1$, $\theta = 30^\circ$, $t/\ell = 0.1$ and $\sigma_{fs}/E_s = 1/100$. Since $m > 0$ for solid cell walls, the failure stresses for brittle crushing in honeycombs with a smaller cell size is higher than those with a larger cell size. However, the cell size effect on the failure surfaces of brittle honeycombs decreases when Weibull modulus becomes larger. Cell size effect is insignificant if Weibull modulus is much larger, for instance $m = 100$ in Fig. 10, as expected for ductile honeycombs.

From Equations 22 and 23, it is found that the failure surfaces for fast brittle fracture in brittle honeycombs depend on the Weibull modulus of solid cell walls. The failure stresses for fast brittle fracture increases with decreasing cell size if m is smaller than 4; Fig. 7 shows the trend for the case of $m = 2$. When $m = 4$, there is no cell size effect on the failure surfaces for fast brittle fracture as shown in Fig. 8. However, the failure stresses for fast brittle fracture increases with increasing cell size for brittle honeycombs with $m > 4$ as shown in Fig. 9 for the case of $m = 8$. It can be expected from Equations 22 and 23 that the failure stresses for fast brittle fracture in ductile honeycombs increases with the square root of their cell size as illustrated in Fig. 10 for the case of $m = 100$.

5. Conclusions

The existing model for the failure envelopes of honeycombs under in-plane biaxial loading has been modified to take into account the effect of variability in the cell-wall modulus of rupture. The variability in the cell-wall modulus of rupture has been accounted for by assuming that it follows a Weibull distribution. The Weibull analysis suggests that the cell-wall modulus of rupture depends on the volume and Weibull modulus of solid cell walls, and the prescribed survival probability. The cell-wall modulus of rupture increases with decreasing prescribed survival probability, relative density and cell size, leading to a cell size effect for the failure envelopes of brittle honeycombs.

In addition, the failure envelopes for brittle honeycombs under in-plane biaxial loading are presented for various prescribed survival probabilities and Weibull moduli. It is found that the areas contained within the failure envelopes for brittle honeycombs increase with decreasing prescribed survival probability. The failure stresses for brittle honeycombs with a smaller Weibull modulus will scatter more widely than those with a larger Weibull modulus. As the Weibull modulus approaching infinity, the failure envelopes come closer to a set of intersecting lines as suggested by the existing model for ductile honeycombs.

Cell size effect is significant for brittle crushing and fast brittle fracture in brittle honeycombs. The failure stresses for brittle crushing increase with decreasing cell size. The failure stresses for fast brittle fracture decrease with increasing cell size if the Weibull modulus, m, of the cell wall material is less than 4; if $m = 4$, there is no cell size effect; and if m larger than 4, the failure stresses increase with increasing cell size. Therefore, the cell size effect on the failure envelopes of brittle honeycombs under in-plane biaxial loading is important and should be taken into account in microstructural design and material selection for brittle honeycombs.

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